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The proposal for Grant AFOSR-88-0188 consisted of the following parts: 1) Modeling complex repairable reliability systems in random environments. 2) Characterization of multidimensional stochastic orderings of positive dependence. 3) Study of the validity of imperfect repair models. 4) Stochastic convexity and reliability system design. We have achieved most of the objectives of the proposal by 31 July 1990.

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FINAL TECHNICAL REPORT

Grant No: AFOSR-88-0188

Title: Reliability modeling of repairable systems in random environments using multivariate conditional failure rates.

Due date: 31 July 1990

The proposal for Grant AFOSR-88-0188 consisted of the following parts:

(1) Modeling complex repairable reliability systems in random environments.

(2) Characterization of multidimensional stochastic orderings of positive dependence.

(3) Study of the validity of imperfect repair models.

(4) Stochastic convexity and reliability system design.

We have achieved most of the objectives of the proposal by 31 July 1990 as is described below.

(1) Consider a multiunit system consisting of components which are repairable. The failure rate of each component depends on various factors. Similarly, the repair completion rate depends on various factors. We have looked specifically on the influence of the identities and ages of the working components on these rates. We also considered the influence of the repair durations on these rates. A graduate student, Mr. Zhu, has been working on these problems, at the University of Arizona, during the last two years under the guidance of the PI's. He has obtained some interesting results. First he discussed a discrete time system with one repairable component. He has proven the monotonicity of the performance of the system as a function of the discrete hazard and repair completion rates: Under a mild technical condition it is true that the smaller is the failure rate and that the larger is the repair completion rate then the larger is the fraction of time (in a stochastic sense) that the system is up (that is, working). Dr. Zhu then extended this result to the multiunit case. In particularly, he showed that when the failure rates and the repair completion rates are conditionally independent (given the history of the system) then a monotonicity result, similar to the univariate case, is still true. The dissertation of Mr. Zhu is still to be written, but the above results will consist of a major part of it.

(2) The PI's have already introduced several notions of stochastic orderings of vectors of lifetimes which have an intuitive meaning in the context of reliability theory. In [1] the PI's have used these orderings in order to characterize multivariate aging properties of dependent random lifetimes. The properties of IFR (increasing failure rate) and PF_2 (Polya frequency of order 2) are

of use and importance in the study of univariate random lifetimes in reliability theory. In [1] these notions are extended to the multivariate setting. The extensions are not merely technical, but they are also meaningful and dynamic. Classes of multivariate IFR and PF_2 distributions are defined and their properties are described. The interrelationship between these classes is given. A graduate student, Mr. Rocha, is now working on these problems at the University of Arizona under the guidance of the PI's. Mr. Rocha has already come up with a notion of dynamic multivariate IFR definition for discrete time dependent random lifetimes. The properties of this notion are similar to the properties of the continuous time analog in [1].

(3) The validity of several models of imperfect repair have been studied by Mr. Valdez-Torres who has been a graduate student at the University of Arizona, and by Mr. Rocha. Their work has been guided by the PI's and a part of their work deals with models of imperfect repair in discrete time. There are more than just one way of defining imperfect repair for discrete univariate random lifetimes. For example, if one item is minimally repaired at time t, does it have another chance of performing its function at time t? Or does it only get a chance to continue its performance from time t + 1 onwards? The former case was considered in the doctoral dissertation of Mr. Valdez-Torres [2] and in the subsequent papers [3] and [4]. The latter case is being studied now by Mr. Rocha. Also the more complex case of multivariate imperfect repair has been studied in [2], [3] and [4] and by Mr. Rocha. The modeling of imperfect repair is closely related to the characterization of multivariate life distributions by the associated multivariate conditional hazard rate functions. All that has to be done is to modify these hazard rate functions. Therefore, it is of importance to simulate distributions of dependent lifetimes by use of these hazard rate functions. A procedure for doing it is described in [5].

(4) Stochastic convexity is closely related to stochastic ordering and inequalities. In order to obtain stochastic ordering one may construct two random variables on the same probability spaces whereas in order to obtain stochastic convexity four such random variables are to be constructed. In [6] the PI survey their work in this area which was partly supported by Grant AFOSR-88-0188. During the period covered by this report, the PI's worked on a study of stochastic ordering which will lead them to further study of stochastic convexity. Several aspects of stochastic ordering were studied in [7] and [8]. In [7] the meaning of the stochastic ordering relation is studied when the random vectors which are compared are assumed to be interchangeable. It is shown that in order to establish the stochastic ordering relation between two such random vectors it is enough to consider only upper sets which are symmetric, rather than all upper sets. In [8] variations of the following result are obtained: If two random vectors are stochastically ordered and have the same marginal distributions then they must have the same joint distribution. For example, it is shown that, for nonnegative random vectors, only the stochastic ordering of positive linear combinations of the random variables is needed in order to obtain stochastic equality.

Some work that has not been described in the proposal has also been done under this grant. In [9] it is shown that a wear process with a general state space, obtained from a fairly general shock model is a strongly IFRA (IFR average) process.

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